Intermittent particle distribution in synthetic free-surface turbulent flows

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Tracer particles on the surface of a turbulent flow have a very intermittent distribution. This preferential concentration effect is studied in a two-dimensional synthetic compressible flow, both in the inertial (self-similar) and in the dissipative (smooth) range of scales, as a function of the compressibility $\mathcal{C}$. The second moment of the concentration coarse grained over a scale r, $\langle n_r^2\rangle$, behaves as a power law in both the inertial and the dissipative ranges of scale, with two different exponents. The shapes of the probability distribution functions of the coarse-grained density n_r vary as a function of scale r and of compressibility $\mathcal{C}$ through the combination $\mathcal{C}/r^{\kappa}$ ($\kappa \simeq 0.5$), corresponding to the compressibility, coarse grained over a domain of scale r, averaged over Lagrangian trajectories.

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I. INTRODUCTION

How turbulence advects particles, a problem with many practical implications, depends in a subtle way on the flow properties. The development of novel experimental techniques, permitting the motion of individual particles to be followed precisely, and of new theoretical concepts has led recently to promising perspectives in this field.

We consider here the dispersion of Lagrangian particles, advected by a two-dimensional surface flow. Even in an incompressible fluid, the motion restricted to the surface is compressible. As a result, particles are observed to concentrate very inhomogeneously. This phenomenon exhibits a number of similarities with the "preferential concentration" of inertial particles in a turbulent flow, where compressibility originates from the fact that particles do not exactly follow the flow.

The phenomenon has been studied experimentally in a laboratory flow at moderate Reynolds number ($R_\lambda \sim 100$), as well as in direct numerical simulations (DNSs) of a turbulent flow with a free surface [1,2]. In these flows, it is convenient to define the compressibility $\mathcal{C}$ at the surface, defined geometrically by the plane $z=0$, by

$$\mathcal{C} = \frac{\langle(\boldsymbol{\nabla}\cdot\mathbf{u})^2\rangle}{\langle(\partial_x u_x)^2 + (\partial_x u_y)^2 + (\partial_y u_x)^2 + (\partial_y u_y)^2\rangle}. \tag{1}$$

This quantity is found to be close to 1/2, both in experiments and in numerics.

The aim of this work is to investigate in a model flow problem some of the fundamental issues regarding the inhomogeneous distribution of particles. To this end, we consider a simplified compressible synthetic flow, obtained by superposing a finite number of Fourier modes, spanning the 'inertial range' of scales: $1/L \lesssim k \lesssim 1/\eta$, with a standard $k^{-5/3}$ velocity spectrum, and with a characteristic time $\tau_k \propto k^{2/3}$, as expected by Kolmogorov theory. The ratio between the largest (integral) scale L and the smallest (Kolmogorov) scale η is a free parameter of the flow, related to the Reynolds number in real flows by $\mathrm{Re} \propto (L/\eta)^{4/3}$. The other free parameter in the flow is the compressibility $\mathcal{C}$ defined by Eq. (1). In our approach these two parameters can be varied at will.

The main focus of this paper is on the properties of the distributions of particles in a synthetic compressible flow. Recent work on preferential concentration of inertial particles has led to the conclusion that the coarse-grained concentration field at scale r, n_r, has very different properties, depending on the range of scale. At very small scales (dissipative range, $r < \eta$), the flow is smooth and the particles accumulate on a multifractal set [3–6]. The second-order moment of the distribution can be estimated in various limits, in particular in the small-Stokes-number limit [7–9], of particular relevance to a number of problems of atmospheric (cloud) physics. The concentration field of inertial particles in the inertial range displays interesting properties [10]. No particular scaling range is found for the second-order moment $\langle n_r^2\rangle$. Interestingly, however, the probability distribution function of n_r is controlled only by the coarse-grained contraction rate.

In a compressible two-dimensional flow, we find that $\langle n_r^2\rangle$ has a power law as a function of scale r, $\langle n_r^2\rangle \propto (L/r)^{\alpha}$, both in the inertial range, with an exponent α_i, and at small scales ($r < \eta$), with an exponent α_d. The exponent in the inertial range is smaller than that in the dissipative range, $\alpha_d > \alpha_i$. The probability distribution of the coarse-grained concentration has algebraic tails at low values of the concentration. These tails reflect the probability that large regions contain very few particles, in qualitative agreement with the observations. Finally, we find that the probability distributions of n_r superpose, provided the ratio $\mathcal{C}(L/r)^{\kappa}$ is kept constant, where κ is an exponent whose value is close to 0.5.

In Sec. II, we discuss the synthetic flow used in this work. Elementary properties of the resulting particle distribution are discussed in Sec. III. The scaling aspects of the distribution are presented in Sec. IV. Section V is devoted to the probability distribution functions of the coarse-grained concentration. Finally, we discuss our results and present our conclusions in Sec. VI.

II. SYNTHETIC FLOW

Instead of using a genuine solution of the Navier-Stokes equations, we consider a synthetic velocity field in two dimensions. The method is known as kinematic simulation (KS), and was developed initially by Fung *et al.* [11], and applied since to many dispersion problems [12–14].

Effectively, the velocity field $\mathbf{v}(\mathbf{x},t)$ is constructed as the sum of an incompressible and a compressible component,

$$\mathbf{v} = \mathbf{v}_I + \mathbf{v}_C, \tag{2}$$

where $\mathbf{v}_I$ ($\mathbf{v}_C$) are the incompressible (compressible) components of the velocity field. These fields are obtained as the superposition of a finite number of Fourier modes:

$$\mathbf{v}_I(\mathbf{x},t) = \sum_{n=1}^{N_k} \mathbf{I}_n \sin(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t + \varphi_n),$$

$$\mathbf{v}_C(\mathbf{x},t) = \sum_{n=1}^{N_k} \mathbf{C}_n \sin(\mathbf{k}_n \cdot \mathbf{x} + \omega_n t + \psi_n), \tag{3}$$

where N_k is the number of modes in the simulation, and the vectors $\mathbf{k}_n$, $\mathbf{I}_n$, and $\mathbf{C}_n$ are given by

$$\mathbf{k}_n = k_n \hat{\mathbf{k}}_n, \quad \mathbf{C}_n = A_n \mathcal{C}^{1/2} \hat{\mathbf{k}}_n, \quad \mathbf{I}_n = A_n (1-\mathcal{C})^{1/2} \hat{\mathbf{k}}_{n\perp} \tag{4}$$

with

$$\hat{\mathbf{k}}_n = \cos(\phi_n)\hat{\mathbf{x}} + \sin(\phi_n)\hat{\mathbf{y}},$$

$$\hat{\mathbf{k}}_{n\perp} = -\sin(\phi_n)\hat{\mathbf{x}} + \cos(\phi_n)\hat{\mathbf{y}}. \tag{5}$$

In Eqs. (3) and (5) the phases φ_n, ψ_n and the angle ϕ_n are randomly distributed in the interval $[0, 2\pi]$ and uncorrelated with each other. The wave numbers $\mathbf{k}_n$ have a norm chosen of the form $k_n = k_1 b^{n-1}$, the parameter b being chosen to be $b = (L/\eta)^{1/(N_k-1)}$. The large (integral) scale L and the small (Kolmogorov) scale η of the flow verify: $L = 2\pi/k_1$ and $\eta = 2\pi/k_{N_k}$. The positive amplitudes A_n of the modes are chosen according to $A_n^2 = E(k_n)\Delta k_n$, where $E(k_n)$ is the energy spectrum of the flow taken as the standard Kolmogorov form $E(k_n) \sim k_n^{-5/3}$. $\Delta k_n = (k_{n+1} - k_{n-1})/2$ for $2 \le n_k \le N_k - 1$, $\Delta k_1 = (k_2 - k_1)/2$, and $\Delta k_{N_k} = (k_{N_k} - k_{N_k-1})/2$. Last, the frequencies ω_n are taken to be $\omega_n = \lambda\sqrt{k_n^3 E(k_n)}$, where λ is a dimensionless parameter, *a priori* of order 1 (we take here $\lambda = 0.5$).

To determine the distribution of particles, we simply integrate the equations of evolution given by the set of differential equations

$$\frac{d}{dt}\mathbf{X}(\mathbf{x}_0,t) = \mathbf{v}(\mathbf{x} = \mathbf{X},t), \tag{6}$$

where $\mathbf{v}(\mathbf{x},t)$ is the synthetic velocity field, defined by Eq. (3), with the initial condition $\mathbf{X}(\mathbf{x}_0,0) = \mathbf{x}_0$. The particles are initially distributed uniformly throughout a square periodic box $(2\pi \times 2\pi)$.

The KS velocity field is only a model of the flow, whose properties reproduce only approximately those of the real flow. In particular, it completely misses the intermittency properties found in DNSs [15]. Recent work on the Kraichnan model has shown that models with nonintermittent flow properties can capture the most important aspects of the particle distribution [16,17]. The fact that the KS flow does not properly describe the sweeping of the small scales by the large scales of the flow is potentially a more serious limitation [18]. It is nonetheless of interest to vary the main free parameter of the flow, the compressibility, and investigate the particle distribution as a function of $\mathcal{C}$.

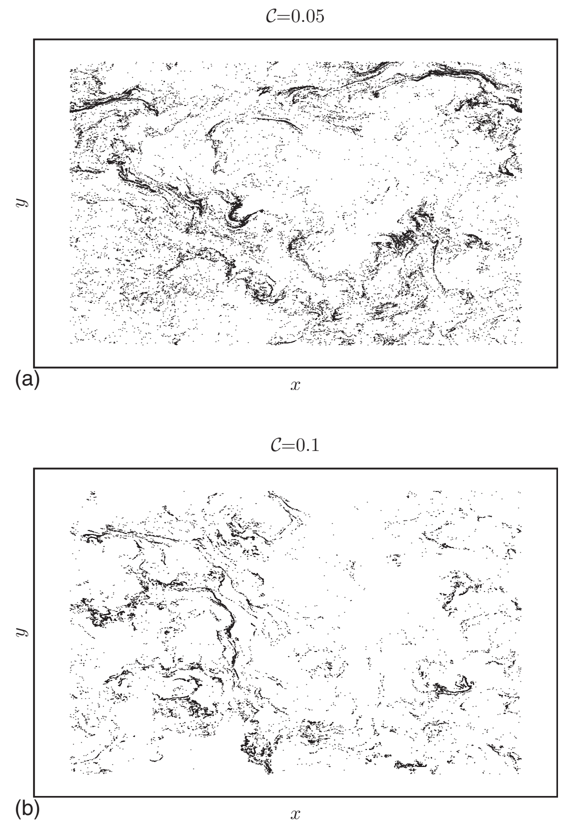

FIG. 1. Visualization of the particle distribution in a box of size $(2\pi \times 2\pi)$, in the stationary state at $t = 4\tau_L$, where τ_L is the integral time scale. The particles are distributed along a nontrivial spatial subset, with a very sparse (intermittent) structure, with many empty regions. As the compressibility increases, the structures become increasingly spotty, as revealed by the comparison between the values of $\mathcal{C} = 0.05$ (top) and 0.1 (bottom).

III. ELEMENTARY PROPERTIES OF THE DISTRIBUTION

In this section, we discuss the elementary properties of the particle distribution as a function of the parameters of the flow, in particular of the compressibility $\mathcal{C}$. A typical example of the particle distribution in the flow is shown in Fig. 1. The distribution corresponding to a compressibility $\mathcal{C} = 0$ is uniform. As $\mathcal{C}$ increases, the particles get concentrated on a sparser subset of the physical space, as a visual comparison between the two panels of Fig. 1 corresponding to $\mathcal{C} = 0.05$ and 0.1 reveals.

The existence of a nontrivial distribution of particles in a compressible flow simply results from the properties of the dynamical system given by Eq. (6) [19–21]. In fact, when the dynamical system is chaotic, that is, provided it has a positive Lyapunov exponent, the particles concentrate along a nontrivial (multifractal) set. Thus, whether the particles have an intermittent distribution depends on the existence of a

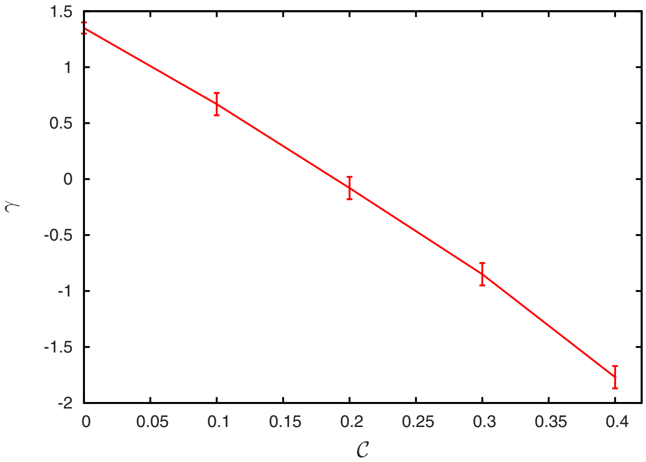

FIG. 2. (Color online) The largest Lyapunov exponent as a function of the compressibility $\mathcal{C}$ of the flow.

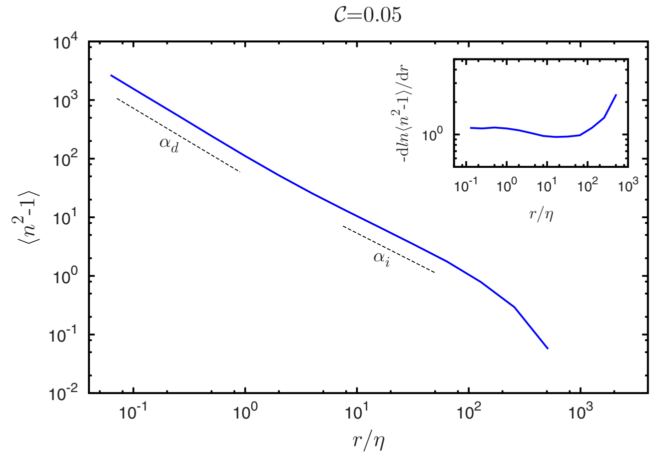

FIG. 3. (Color online) Deviation of the second moment of the coarse-grained particle concentration from 1, the value at $r=L$, as a function of r. Two different power laws, indicated by the two dashed lines, are observed, one in the inertial range ($r>\eta$) with an exponent α_i, and one in the dissipative range $r<\eta$, with an exponent α_d. The inset shows the logarithmic derivative; the scaling regions are clearly shown by the plateaus. The parameters used in this calculation are $L/\eta=1024$, and $\mathcal{C}=0.05$.

positive Lyapunov exponent for the dynamical system given by Eq. (6). The largest Lyapunov exponent for the flow considered here has been computed numerically using standard methods, and the result is shown in Fig. 2. The largest Lyapunov exponent, denoted γ, decreases linearly when $\mathcal{C}$ increases, and becomes negative for $\mathcal{C}\approx 0.19$.

The range of values of $\mathcal{C}$ over which the flow is chaotic is significantly smaller than the corresponding range in other models of turbulent flows, such as the Kraichnan model, or its elaborations with finite correlation times [22]. We interpret the strong concentration in our synthetic system as a result of the simplified structure of the flow at a given scale (only one wave number $\mathbf{k}_n$ instead of a full shell); see Eq. (3). This effect, however, is not a serious concern for our investigation of issues related to preferential concentrations.

IV. PREFERENTIAL CONCENTRATION AND SCALING EXPONENTS

Figure 1 reveals that particles concentrate over a set with a multiscale structure. Various diagnostics have been proposed to characterize the scale distribution of the set where particles accumulate. We focus here on the moments of the Eulerian particle density (computed at fixed spatial location) coarse grained over a square of size r.

Effectively, the computational domain, of size $(2\pi)^2$, is decomposed into 2^{2p} squares of size $r=2\pi/2^p$. The number of particles in a given square, divided by the area of a square, $4\pi^2/2^{2p}$, defines the density of particles in this square. The overall concentration of particles being immaterial, we define n_r as the concentration in a small square, divided by the mean concentration, so that the mean value of n_r is equal to $\langle n_r\rangle=1$. The larger the value of $\langle n_r^2\rangle-1$, the larger the fluctuations of the particle concentrations. The dependence of $\langle n_r^2\rangle-1$ on the scale r thus characterizes the importance of the lack of homogeneity of the particle density at scale r, and provides a good measure of the heterogeneity of the concentration. It is easy to show that, for a homogeneous distribution of particles, $\langle n_r^2\rangle$ is independent of r. When particles accumulate along a line, $\langle n_r^2\rangle \propto r^{-1}$, and when particles accumulate on a single point, $\langle n_r^2\rangle \propto r^{-2}$.

The second moment of the coarse-grained particle concentration is shown in Fig. 3 for a run with $\mathcal{C}=0.05$ and $L/\eta=1024$. Figure 3 shows that $\langle(n_r^2-1)\rangle$ behaves as a power law in the dissipative range (for $r<\eta$), characterized by the exponent α_d: $\langle n_r^2\rangle \propto (\eta/r)^{\alpha_d}$. Similarly, in the inertial range $\eta<r<L$ ($L/\eta=256$ in the present calculation), the deviation of $\langle n_r^2\rangle$ from 1, the value at $r=2\pi$, behaves as a power law: $\langle(n_r^2-1)\rangle \propto (L/r)^{\alpha_i}$. The second moment of the coarse-grained particle distribution over a scale r is thus characterized by two different exponents, one in the inertial, α_i, and one in the dissipative range, α_d. It is found that $\alpha_d>\alpha_i$, which implies that the heterogeneities grow faster in the dissipative than in the inertial range, an effect already known in the context of preferential concentration of heavy particles in turbulent flows.

Comparison of several runs with the same value of the compressibility but different values of the ratio L/η did not reveal any significant variation of the values of the exponents $\alpha_{i,d}$. The exponents $\alpha_{i,d}$, however, do depend systematically on the compressibility $\mathcal{C}$. The dependence is shown in Fig. 4. The systematic measurement of the exponents $\alpha_{i,d}$, shown in Fig. 4, has been carried out in a flow with an inertial range of $L/\eta=256$. The exponents $\alpha_{i,d}$ increase monotonically with $\mathcal{C}$, the value of α_d being always larger than α_i. The small values of $\alpha_{i,d}$ are difficult to compute with accuracy. Still, our data provide evidence that the values of $\alpha_{i,d}$ both go to zero as $\alpha_{i,d}\propto \mathcal{C}^{1/2}$ when $\mathcal{C}\to 0$, at least over the range of values of $\mathcal{C}$ we could investigate; see the inset of Fig. 4.

The statistical properties of the particle distribution have been shown to be multifractal [19]. Our own numerical results are consistent with these findings.

The existence of a power-law dependence for $\langle n_r^2\rangle$ in the inertial range contrasts with what has been observed for the case of preferential concentration of heavy particles in a turbulent flow. In this case, no obvious scaling law has been

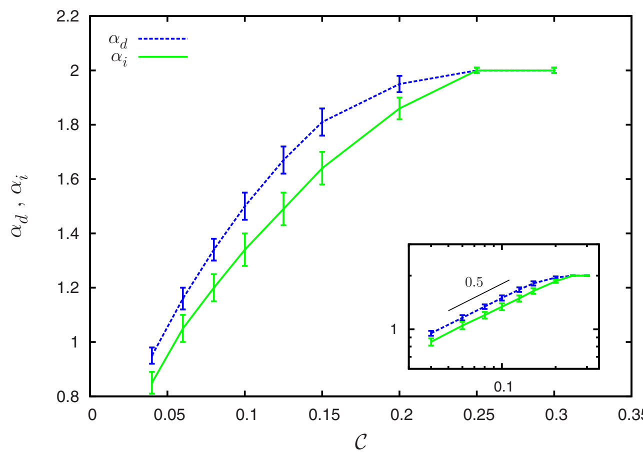

FIG. 4. (Color online) Values of the exponents α_i and α_d as a function of the compressibility $\mathcal{C}$. The exponents increase monotonically with $\mathcal{C}$. It is found that the exponents tend to 0 as $\mathcal{C}\to 0$, with a power law $\alpha_{i,d}\propto\mathcal{C}^{1/2}$; see the inset.

observed [10]. In our problem, the existence of a power-law dependence has been predicted by Fouxon [26].

V. PROBABILITY DISTRIBUTION OF COARSE-GRAINED CONCENTRATION

In this section, we discuss the probability distribution function (PDF) of the coarse-grained particle distribution over a scale r. One of the motivations comes from the recent numerical suggestion that, in the problem of preferential concentration of heavy particles, the fluctuations of the coarse-grained particle distribution have a distribution whose shape depends only on the effective compressibility at scale r [10].

The PDF of the coarse-grained particle concentration n_r is shown in Fig. 5 as a function of the compressibility at a scale $r=L/2^5$, well within the inertial range ($L/\eta=2^8$ in the flow considered). Consistent with the notion that the fluctuations around the mean value increase when the value of $\mathcal{C}$ increases, Fig. 5 demonstrates that the distributions get broader as the compressibility $\mathcal{C}$ increases. Interestingly, the probability distribution functions of n_r are qualitatively very similar to the corresponding PDFs for the problem of preferential concentration of heavy particles [10]. At low values of n_r, the PDF exhibits a power-law decay: $P(n_r)\propto n_r^{(\xi-1)}$. The exponent ξ depends on both the ratio L/r and the compressibility $\mathcal{C}$. The value of ξ is shown in Fig. 6. The slow decay of the PDF at small values of n_r reflects the fact that there are large regions without many particles, even at scales r within the inertial range. In this respect, the decreasing value of ξ as $\mathcal{C}$ increases reflects the fact, visible in Fig. 1, that the empty regions of space are getting bigger and more probable as the compressibility becomes larger.

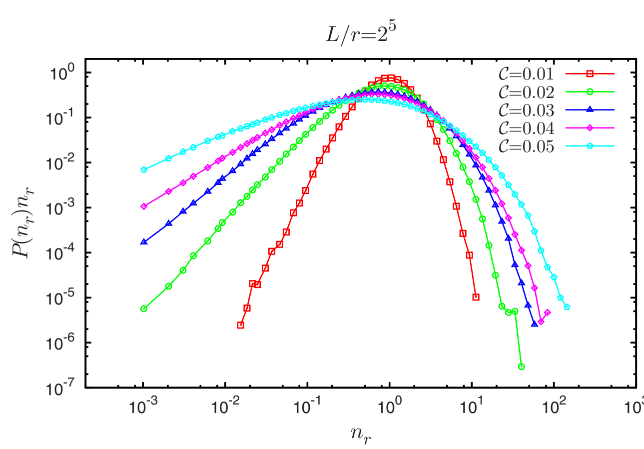

FIG. 5. (Color online) PDF of the coarse-grained particle concentration over a scale r within the inertial range: $L/r=2^5$. As the compressibility $\mathcal{C}$ increases, the fluctuations grow.

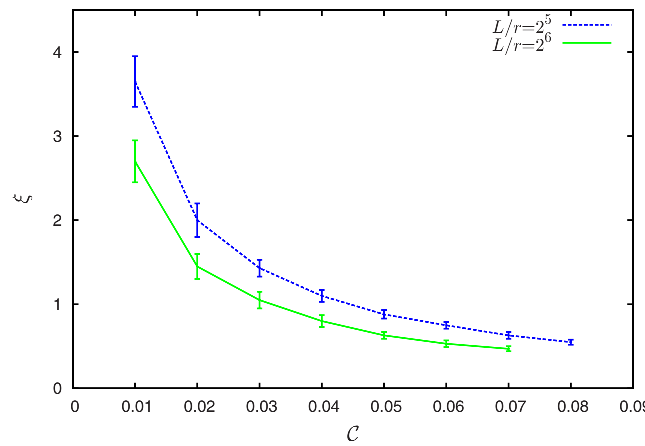

FIG. 6. (Color online) Exponent ξ characterizing the decay of the PDF of n_r when $n_r\to 0$: $P(n_r)\propto n_r^{(\xi-1)}$. As the compressibility increases, or as the scale decreases, the fluctuations become wider and the exponent ξ decreases to a very small value.

As has been found in the problem of preferential concentration of heavy particles, the PDFs of the coarse-grained concentration can be superposed provided a simple relation exists between the compressibility $\mathcal{C}$ and the scale r. Accordingly, we found that, provided $\mathcal{C}(L/r)^\kappa$ is constant ($\kappa\simeq 0.5$), the PDFs superpose very well, especially at small concentrations $n_r\ll 1$, as demonstrated by Fig. 7.

Our numerical results demonstrate that the shape of the PDF of n_r depends only on $\mathcal{C}/r^\kappa$ at small concentrations. As a result, the exponent ξ that characterizes the power-law decay of the PDF at low values of n_r is expected to be a function of $\mathcal{C}/r^\kappa$. In fact, Fig. 8 indicates that, over the range of values covered by the present study, the exponent ξ has a power-law dependence on $\mathcal{C}/r^{0.5}$: $\xi\propto(\mathcal{C}/r^{0.5})^{-\beta}$, with $\beta\approx 0.9$.

The observations shown in this section are very reminiscent of the results obtained in [10]. There, it was shown that the probability distributions of the coarse-grained particle density n_r have a shape very similar to the one shown in Fig. 5. In addition, the PDFs were found to superpose, provided the product St $r^{-5/3}$, where St is the Stokes number, is constant. The latter was then interpreted as resulting from similar properties of the compression rate related to the pressure Hessian, coarse grained over a scale n_r.

With this motivation, we have computed directly the compressibility field $\boldsymbol{\nabla}\cdot\mathbf{v}$, coarse grained over a scale r, at several values of the scale r. The coarse graining has been carried out by applying a Gaussian filter $\mathcal{G}_r(\mathbf{x},\mathbf{x_0})$

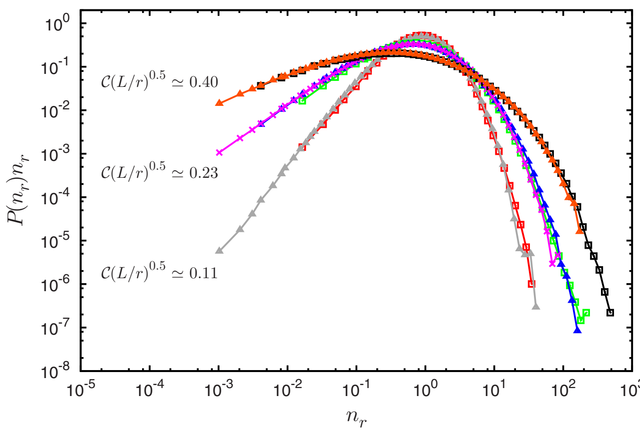

FIG. 7. (Color online) Superposition of the PDFs of the coarse-grained concentration n_r in the inertial range for three different values of the ratio $\mathcal{C}(L/r)^{0.5}$, as indicated in the figure. For $\mathcal{C}(L/r)^{0.5} \simeq 0.11$ the two PDFs correspond to $\mathcal{C}=0.01, 0.02$, and $L/r=2^7, 2^5$, respectively. For $\mathcal{C}(L/r)^{0.5} \simeq 0.23$ the three PDFs correspond to $\mathcal{C}=0.02, 0.028, 0.04$ and $L/r=2^7, 2^6, 2^5$. For $\mathcal{C}(L/r)^{0.5} \simeq 0.40$ the two PDFs correspond to $\mathcal{C}=0.05, 0.07$ and $r=2^6, 2^5$.

$\equiv \frac{1}{2\pi r^2}\exp[-(\mathbf{x}-\mathbf{x_0})^2/(2r^2)]$ to the compressibility field:

$$(\nabla \cdot \mathbf{v})_r(\mathbf{x_0}) = \int d^2\mathbf{x}\ \mathcal{G}_r(\mathbf{x},\mathbf{x_0})\ \nabla \cdot \mathbf{v}(\mathbf{x}). \tag{7}$$

The resulting coarse-grained compressibility $(\nabla\cdot\mathbf{v})_r$ was then averaged over many Lagrangian trajectories. It was found that, at long times, the averaged value tends to a limit $\langle(\nabla\cdot\mathbf{v})_r\rangle_{\mathcal{L}}$, which is shown in Fig. 9 as a function of r for several values of $\mathcal{C}$. The value of $\langle(\nabla\cdot\mathbf{v})_r\rangle_{\mathcal{L}}$ is found to be negative, and proportional to $\mathcal{C}$. The negative value indicates that particles accumulate in regions where the flow is locally compressible (the Eulerian average of the compressibility of the flow is zero). In addition, Fig. 9 reveals that the compressibility behaves essentially as

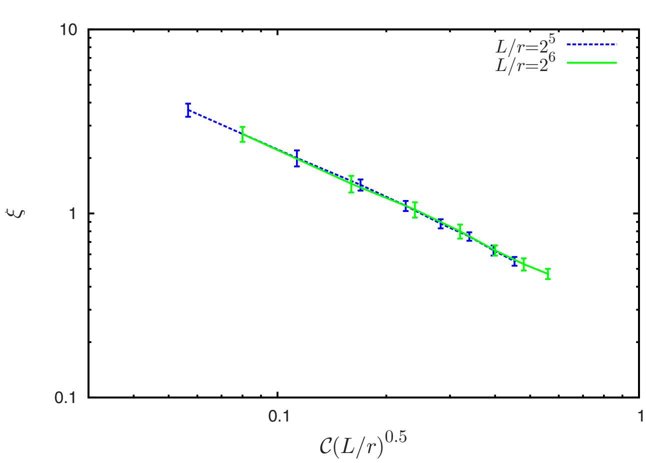

FIG. 8. (Color online) Exponent ξ that determines the power-law decay of the PDF of n_r when $n_r \to 0$ as a function of $\mathcal{C}/r^{0.5}$. The dependence can be well approximated by a power law $\xi = A(r^{0.5}/\mathcal{C})^{\beta}$, with $\beta \approx 0.9$.

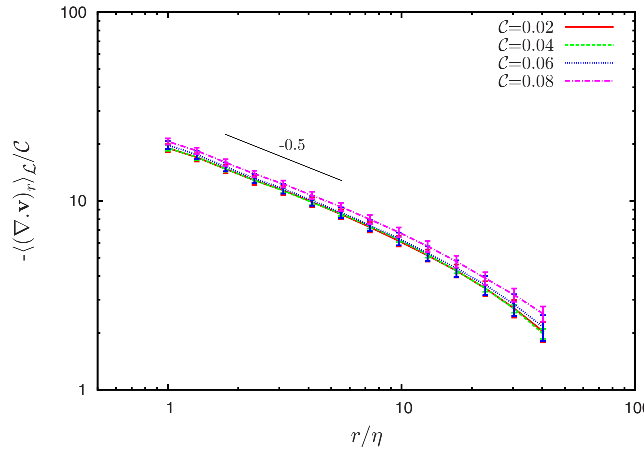

FIG. 9. (Color online) Long-time limit of the compressibility along Lagrangian trajectories, and coarse grained over a domain of scale r. The values of the compressibility are rescaled by the parameter $\mathcal{C}$. As a function of r, $\langle(\nabla\cdot\mathbf{v})_r\rangle_{\mathcal{L}}$ has a power-law behavior $\langle(\nabla\cdot\mathbf{v})_r\rangle_{\mathcal{L}} \propto r^{-\kappa}$, with the exponent κ close to 0.5. A similar scaling behavior has also been found at $L/\eta=1024$.

$$\langle(\nabla \cdot \mathbf{v})_r\rangle_{\mathcal{L}} \propto -\mathcal{C}r^{-\kappa}, \quad \kappa \approx 0.5, \tag{8}$$

when r is in the inertial range, for $r/L \lesssim 0.1$.

This observation provides an explanation for the fact that the shape of the PDFs of the coarse-grained particle density n_r depends on the reduced parameter $\mathcal{C}/r^{\kappa}$ only, at least when n_r is in the inertial range. Together with Eq. (8), the reasonable assumption that the properties of the distribution of particle, coarse grained over a size n_r, depend on the coarse-grained compressibility at scale n_r, as suggested in Ref. [10], leads to the expectation that the distribution of n_r depends on $\mathcal{C}/r^{\kappa}$, as seen in Fig. 7.

Although seemingly simple, the exponent $\kappa \approx 0.5$ does not appear to have an obvious explanation. Indeed, the naive analysis based on a dimensional argument would suggest instead $\langle(\nabla\cdot\mathbf{v})_r\rangle_{\mathcal{L}} \propto r^{-2/3}$. However, the coarse-grained value of the compressibility along Lagrangian trajectories depends on many effects, which cannot be captured by a casual argument; hence the deviation of κ with respect to the "naive" exponent 2/3 (see Fig. 9).

VI. DISCUSSION AND CONCLUSIONS

We have studied in this paper the statistical properties of the distribution of particles advected by a two-dimensional compressible flow, a problem recently investigated experimentally [1] and numerically [2] in the context of a free-surface turbulent flow. We have considered a synthetic flow, which differs quantitatively in a number of ways from the real surface flow in Ref. [1]. The synthetic flow has a well-defined inertial range of scale, between a large, inertial scale L and a small, dissipative (Kolmogorov) scale η, the ratio L/η being a measure of the Reynolds number. For our own purpose, the most significant parameter characterizing the flow is the compressibility $\mathcal{C}$ defined by Eq. (1). The intermittency effects seen in real turbulent flows [15] are not properly accounted for.

From a theoretical point of view, the statistical properties of the particle distribution are well understood in the dissipative range. For $r \ll \eta$, the flow is smooth and the particle distribution has been shown to have a multifractal measure [19]. The understanding of the properties of the particle distribution in the inertial range of scales is still incomplete, despite recent progress [10,23].

One of our main results concerns the variations of the second moment of the coarse-grained particle density over scale r as a function of r. We found a power-law dependence $\langle n_r^2 \rangle \propto (L/r)^{\alpha_{i,d}}$, where the exponent α_i (α_d) characterize the distribution in the inertial (dissipative) range of scales. For the related problem of heavy particle dispersion by a turbulent flow, the power-law dependence had been clearly established in the dissipative range of scales. This is to be contrasted with the inertial range of scales, where no power law had been seen numerically [10]. The exponents are found to satisfy $\alpha_i \leq \alpha_d$, and to vanish when $\mathcal{C} \to 0$ as $\alpha_{i,d} \sim \mathcal{C}^{1/2}$.

The observation that the shape of the PDFs of the coarse-grained particle distribution depends only on the reduced parameter $\mathcal{C}/r^{0.5}$, for r in the inertial range ($\eta \leq r \leq L$), is the other major result of this work. A qualitatively similar result was obtained for the problem of heavy particles advected by a turbulent flow [10]. The numerical results also demonstrated that the coarse-grained compressibility over a scale r, $(\nabla \cdot \mathbf{v})_r$, averaged over the many Lagrangian particles, is on average equal to $\mathcal{C}(r/L)^{-\kappa}$, with $\kappa \approx 0.5$. Thus the observed similarity between the coarse-grained particle distributions can be related to the property of the coarse-grained compressibility, as suggested in Ref. [10].

The properties of the distribution of particles advected by the (model) compressible surface flow studied here are thus very reminiscent of the properties obtained in the case of heavy particles advected by a turbulent flow—the lack of a simple power law for the second moment of $\langle n_r^2 \rangle$ in the latter case being the most noticeable difference between the two cases. These results may be pointing to a simple and universal mechanism explaining these phenomena, perhaps in the spirit of Ref. [23]. It would be interesting to investigate other quantities characterizing the clustering of particles, such as entropy production [24,25] as a function of time, with our model flows. Thus, despite its simplicity and the quantitative differences with the realistic surface flow that motivated this work, the simple problem studied here has revealed properties that seem to be in common with other important mixing situations.

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